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Optimization models for military aircraft deployment

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AIRCRAFT DEPLOYMENT

by

Michael C. Puntenney

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Optimization Models for Military Aircraft Deployment

by

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Submitted in partial fulfillment of the requirements for the degree of

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I. INTRODUCTION

In the modern world of rapid mobility, the difference between a successful and an unsuccessful deployment of military units and equipment will depend on the efficient allocation of transportation assets to the requirements which need to be moved within prescribed time intervals. These transportation assets consist primarily of military transport aircraft, ocean-going vessels, and rail and truck vehicles. The movement requirements may consist of any material item or person requiring transportation from one location to another such as an infantry division, petroleum products, heavy equipment, artillery, subsistence items, and many others. Small differences in the timing of delivery or the effective use of assets could determine the difference between an action being won or lost [Ref. 1].

This thesis considers the question of whether a potential military operation can be initiated and sustained from the perspective of air transportation. It is valuable to know what assets will be used, where they will be needed and whether there are bottlenecks in the transportation process which significantly affect the operation. It is not necessary to develop detailed schedules and load plans for aircraft and movement requirements for this initial planning; creating such schedules and plans is beyond the scope of this work. However, the initial planning process for the allocation of transportation assets will need to be sensitive to the timing of delivery of these movement requirements and to the effective use of the transportation assets. The problem of determining the appropriate deployment of aircraft given a set of requirements which must be moved within prescribed time intervals and with specific delivery times which are the most opportune for the objectives of the operation will henceforth be called the **deployment problem**. Past model development has shown that the complexities involved with combining different modes of transportation in a single model require solving an extremely large problem. Consequently, a modular approach which separates the air, land, and sea allocation problems is usually employed [Ref. 2].

The United States Transportation Command (USTRANSCOM), is the Department of Defense's agency which has been most concerned with the deployment problem. This organization was created to consolidate the global air, land, and sea transportation assets and requirements which are used by the military to meet national security objectives. While tasked with coordinating the development of deployment plans involving transportation assets, one of USTRANSCOM's principal endeavors is to provide automated data processing systems to streamline the planning process. The Joint Deployment Agency (JDA), the predecessor organization of USTRANSCOM, recognized the potential of automated data processing to assist with mobilization planning. The Joint Deployment System (JDS), a comprehensive computer planning system initiated under the auspices of the JDA, was designed to plan, execute, and monitor the deployment of units during both peacetime and times of crisis. Many reseach initiatives directly related to the deployment problem have been undertaken to support this system. [Ref. 3]

The first linear programming based effort at modeling and solving the deployment problem was the System for Closure Optimization Planning and Evaluation (SCOPE), a submodule of the JDS [Ref. 4]. This system used a Benders decomposition method to solve a continuous variable model consisting of both sea and air assets over discrete time periods. The formulation separated the model into a master problem involving the assignment of assets to delivery missions and a subproblem involving the assignment of movement requirements to transportation assets. Because the solution method converged slowly or not at all, and because it did not model sea assets integrally, the Benders decomposition approach was considered to be unsuitable for the JDS system [Ref. 5 : p. 20].

The deployment problem for the allocation of sea assets has been studied by many researchers since the original SCOPE model. A continuous variable deployment model for both sea and air assets similar to that of the SCOPE model was undertaken by Collier [Ref. 5] using linear programming and a variable reduction heuristic. This model was developed to assist planners with designing feasible deployment plans and selecting appropriate modes of transportation for movement requirements. An alternative model for the deployment problem involving only sea transportation assets was developed by Lally [Ref. 6] using integer programming. This model avoided the integrality problems which occurred for sea assets in the SCOPE model. While both of these studies demonstrated the ability to use optimization techniques for moderately sized instances of problems, neither of these techniques have been employed in the JDS system. In its current form, the JDS uses simulation-based heuristics which are capable of providing solutions to highly detailed and very large resource allocation models. However, the JDS still has difficulty in providing solutions to large instances of problems in a timely manner [Ref. 7], and the solutions obtained cannot be considered to be optimal with respect to any measure of effectiveness. More promising developments involving optimization techniques for solving large-scale deployment problems involving only sea transportation assets have been made by Buvik [Ref. 8] and Lima [Ref. 9] using set covering models with dynamic column generation.

Another initiative which is being pursued by USTRANSCOM to solve the deployment problem is the development of the Deployment Analysis Prototype (DAP). The DAP is a deployment modeling system developed principally as a technology demonstration implementable entirely on a 80286-based personal computer using many of the same inputs as the JDS. This system is used as a base for the development of modeling techniques which could be used as stand-alone tools for deployment planning as well as a testbed for the development of innovative planning techniques for the JDS. Of specific interest for this thesis will be the continuous variable model designed to solve the deployment problem for air transportation assets which was developed by Rosenthal [Ref. 10] to succeed the original air transportation allocation model in the DAP.

The nature of deployment modeling on a large scale poses many challenges when optimization methodology is used. This thesis develops prototypic planning tools to aid USTRANSCOM with the allocation of air transportation assets in an effective and timely manner. Formulations are developed using the same problem format, but with a different modeling approach, of those previously laid out by Rosenthal for the allocation of air assets. The purpose for developing these new models is to enhance system solution-time performance, model resolution, flexibility, and compatibility with other modeling programs. Three optimization models and an iterative solution algorithm designed to improve the DAP are provided. The models and algorithms form a collection of ideas related to the deployment problem as it relates to air transportation.

Chapter 2 of this thesis provides a general discussion of the aircraft deployment problem as well as the derivation and description of the optimization models and algorithm. The first model is based on the weighted distribution problem, more commonly known as the generalized transportation problem [Ref. 11]. As in earlier models, this model uses continuous variables to represent the assignment of movement requirements to aircraft and the assignment of aircraft to missions. The convention of using continuous variables is justified by the continuous nature of air operations, the aggregate nature of available data and the level of detail required for the preliminary screening of potential operation plans. Unlike the earlier models, this model provides information on the movement requirements which are not deliverable at the opportune time for the operation plan being considered. This information is particularly important for deciding whether sufficient deliveries of particular movement requirements will occur and thus allow the military operation to proceed. Additionally, this model lays the foundation for developing the other models. The second model, a transformation of the generalized transportation formulation, significantly reduces the number of variables when movement requirements exist across most route, opportune delivery period, and cargo type combinations. This model exhibits a more stable model generation and solution structure than the generalized transportation formulation when solved as a linear program.

A solution algorithm which searches for and then solves subproblems over independent time windows of the original time horizon is described. In many instances, this technique may offer the ability to model extended time horizons and solve larger instances of problems.

The DAP, in its current form, uses a post-solution processor which requires integral aircraft solutions. In order to reform the linear programming solution to integers as required by this post-solution processor, the resulting nonintegral solutions of the continuous variable models are converted to integer solutions. This third model uses a greedy rounding model to allocate fractional values making use of any remaining aircraft supplies.

Test results of typical problems are provided in Chapter 3. A typical problem size involves nine routes, 80 movement requirements distributed across two cargo classes involving 200,000 short tons of freight, 250 aircraft per time period using four different aircraft types, with 12 time periods in the time horizon. A summary of the thesis is provided in Chapter 4. All of the models and techniques described in this thesis are programmed in the General Algebraic Modeling System (GAMS) [Ref. 12] using a 80286 based personal computer. GAMS was used because it facilitates rapid changes in the model constraints to reflect unique situations for a particular military operation. GAMS programs for the models described in Chapter 2 are contained in the appendices.

II. PROBLEM FORMULATION

A. GENERAL PROBLEM DESCRIPTION

The military aircraft deployment problem involves the shipment of military units and equipment between ports of embarkation and debarkation within prescribed time intervals using an available pool of aircraft. These shipments are assumed to be unidirectional, going from one geographic area to another. An example of this would be the movement of a rapid deployment force from the east coast of the continental United States to Europe. This chapter presents the derivation of a collection of models and an algorithm which can be used to evaluate the general suitability of preliminary military operation plans which depend on the solution of this problem. The remainder of this section provides a general description of the terminology and elements of the models.

For the models used in this study, the planning horizon is a collection of discrete non-overlapping time intervals called **periods**. The interval of a period depends on the distances to be travelled and the speed of the aircraft. Any cargo shipped in a period will arrive at its destination within that same period. Additionally, the intervals for the different periods may be of equal or varied duration. One reason for allowing the duration of time periods to vary is because of the increasing uncertainty of events and requirements the farther the time horizon is extended into the future. Consequently, the amount of detail required in the model diminishes with time which allows increasing the time interval as periods progress.

The deployment problem involves the transfer of military units and equipment. Any collective group of materials or individuals with a common port of embarkation and debarkation which is requested to be moved within a specific time period is called a **movement requirement**. The period of requested delivery is called an **opportune delivery period** since it reflects the period of delivery which is deemed to be most beneficial for the objectives of the operation plan being considered. In addition to the opportune delivery period, an **interval of permissable periods** is defined which contains those periods for which movement requirements are allowed to be delivered early or late from the opportune delivery period. An **available-to-load period** is also used which indicates the earliest possible period for which a particular movement requirement can be loaded on an aircraft. This period represents the availability of cargo, while the early delivery period mentioned above represents the earliest delivery period allowable which still meets the objectives of the operation.

All items within a particular movement requirement will have a single homogeneous classification with a standard physical unit. For clarity, the definition of the term **physical units** is noted as being quite distinct from the term **military units**. While the term military units refers to an organized body of troops, physical units refers to the fixed quantity of measure for items being transferred. The classifications of movement requirements are typically passengers, bulk cargo, oversized cargo, or any other appropriate grouping, while the physical units of measure are typically tons for equipment and passengers for military units. These movement requirement classes will be called **cargo** for simplicity.

The aircraft which can be deployed during a particular period are not treated individually, but rather as a pool of aircraft available to perform missions. The assumption that deployable assets are pooled without regard to the location of aircraft at the start of a period is reasonable because of the continuous nature of aircraft movements and the assumption that all embarkation ports are in the same geographic region. This simplification significantly reduces the number of variables required in formulating the model since the location of each aircraft does not have to be tracked. Any of the aircraft assigned to the pool may be used on successive tours between a single port of embarkation and debarkation during that same period. These tours are designated by the forward **routes** which connect the port of embarkation with the port of debarkation. Aircraft **missions** are defined to be a route, delivery period, and aircraft type combination which are used to transport various movement requirements. For this study, the aircraft delivery periods are called **mission periods**. Since the deployment problem assumes a unidirectional flow of military units and equipment, aircraft are considered to return empty from ports of debarkation. Aircraft are classified by aircraft-type and have associated movement requirements which are compatible with the aircraft design. The aircraft are assigned from the pool based on the relative importance of the aircraft. One reason for assigning aircraft usage priorities is that certain aircraft are better suited to particular military operations.

The aircraft deployment models of this study use formulations with a channel flow concept similar to that originally proposed in the SCOPE model [Ref. 3]. Channels or routes are defined only on possible pairs of embarkation and debarkation ports where movement requirements exists. The capacity to transport quantities of a requirement over a route using a specific aircraft type is calculable. This **route capacity** is a function of many factors such as the expected aircraft speed, the length of the interval of time in the period, the distance between ports of embarkation and debarkation, the loading capacity of the aircraft, the expected layover time at ports, and the compatibility of aircraft types with particular ports. The route capacity is an input parameter available to the model.

The model formulations employ continuous variables to represent shipments of movement requirements across routes. A shipment assigned to a mission is defined to be delivered entirely within the period of the mission. The quantity of cargo shipped per period on a route using a particular aircraft is determined by its route capacity. As such, shipments are based on the rate of transfer specified by the route capacity during a period and not discrete plane-loads of cargo. The amount of cargo transferred is limited by the total supply of aircraft available. The use of continuous variables is reasonable when nontrivial numbers of aircraft are required on any route during a military operation. For such cases, the continuous solutions should be sufficiently accurate to provide a good preliminary assessment of the logistic supportability of an operation plan. The models are required to be able to not only track the closure of all movement requirements to their debarkation ports, but also those portions of movement requirements which are delivered earlier or later than their opportune delivery period. This is especially important for determining whether a marginal operation plan is supportable. Aircraft supply constraints are imposed on the model by using total route capacity restrictions determined by the total number of a particular aircraft type available during a period. Various linear side constraints can be included to adapt the model to specific operational situations such as capacity limitations at ports or minimum shipment level restrictions for movement requirements.

The objective function consists of costs which are combinations of penalties assigned to the different events associated with transporting movement requirements. The cost scheme for a model includes relative penalties for (a) the nondelivery of movement requirements, (b) the early or late delivery of cargo from the opportune delivery period, and (c) the assignment of cargo and aircraft types. The additive combination of the different classes of penalties creates a cost scheme which prioritizes all possible shipments which could occur using the various movement requirements, aircraft, and delivery periods. The specific cost of a transfer may not be economically meaningful; however, the relative cost of the transfer in relation to the cost of other types of transfers is important. The dominant penalties typically are for delivery deviations from the opportune delivery period and nondelivery of movement requirements. The different types of penalty classes are hierarchically scaled so the dominance of particular penalty classes is preserved when they are added to lower priority penalty classes. While the objective function minimizes the costs derived from these penalties, the general objective of the model is to deliver all movement requirements in a timely manner while efficiently using available aircraft.

B. GENERALIZED TRANSPORTATION FORMULATION

The original formulation by Rosenthal [Ref. 10] for the aircraft deployment problem used dynamic equations to model the shipment of movement requirements and early and late deliveries aggregated by mission delivery period in order to reduce the problem size. However, this model was unable to provide information about the origination of movement requirements which were delivered early or late and did not consider the possibility of an infeasible problem. The models of this thesis take these aspects of the deployment problem into consideration with emphasis being place on the size and solvability of problems. This section presents a model which solves the aircraft deployment problem using a weighted distribution formulation [Ref. 11].

The weighted distribution formulation is a generalized transportation network which weights the variables in a problem with non-unit coefficients. For the aircraft deployment problem, these weighting coefficients are multipliers that represent the route capacities which regulate the transfer rates of movement requirements. Side constraints may be added to the problem to represent specific operational situations. Many of these side constraints may be formulated so they extend the network of this **Generalized Transportation Formulation (GTF)**. The primary index sets for the model are:

$i \in I$	embarkation ports.
$j \in J$	debarkation ports.
$r \in R$	routes.
$a \in A$	aircraft types.
$h \in H$	cargo types.
$t, \tau \in P$	periods of time horizon.

For notational purposes, t and τ are dummy indices representing any period in P. The location of t and τ in a data parameter or variable determines the context for the use of either of these dummy indices. The first index set involving P in a data parameter or variable always relates to a mission period while the second index set involving P always relates to the opportune delivery period. Additionally, the set of routes R, represents the cartesian product of embarkation and debarkation ports.

For simplicity, the data for the problem is expressed using a single measurement system in short-tons (stons), stons/aircraft, etc., which are used in a specific period. Measurement systems using volumetric or other physical units are possible. The data for the problem is:

$MREQ_{r\tau h}$	movement requirement of cargo type h in stons to be shipped over route r with opportune delivery period τ.
$ALP_{r\tau h}$	available-to-load period for a movement requirement.
$\underline{d}_{r\tau h}, \overline{d}_{r\tau h}$	number of periods which a movement requirement may be delivered early or late from its opportune delivery period.
$ASUP_{at}$	number of aircraft of type a available in mission period t.
$RCAP_{art}$	cargo capacity in stons/aircraft for aircraft type a over route r during mission period t.
$PCAP_{it}$	port throughput capacity for movement requirements involving cargo type h in stons for port i during mission period t.
$C^{X}_{ar\tau h}$	cost per ston for shipping cargo type h with opportune delivery period τ over route r using aircraft type a during mission period t.
$C^{Y}_{r\tau h}$	cost per ston for not delivering cargo within a movement requirement having cargo type h with opportune delivery period τ required over route r.

The satisfaction of movement requirements is accomplished using variables associated with available aircraft. The delivery of the movement requirements can be identified as belonging to two different classes of assignment. The first class of assignment represents the direct satisfaction of a movement requirement with aircraft having a mission period the same as the opportune delivery period, henceforth to be called **direct period deliveries**. The second class represents the satisfaction of movement requirements by aircraft with a different mission period than the opportune delivery period (yielding early or late deliveries of cargo) henceforth to be called **cross-period deliveries**. Both of these classes are represented in the variable $X_{art\tau h}$. If there are inadequate supplies of available aircraft to satisfy all movement requirements, variables associated with undelivered cargo are used. The variables for the undelivered movement requirements will be called **infeasibility variables**. These variables bear a sufficiently large cost so there use occurs only when there is no other means of delivery. The variables for the model are:

$X_{a t \tau h}$ stons of cargo type h with opportune delivery period τ shipped over route r using aircraft type a during mission period t .

$N_{r \tau h}$ undelivered stons of a movement requirement having cargo type h with opportune delivery period τ required over route r .

In addition to the primary index sets, the following derived index sets are necessary. The set of periods P in the time horizon must contain the set of periods P_m for which aircraft are available to perform missions, that is $P_m \subset P$. The set of periods P in the time horizon must also contain the set of periods P_o for the opportune delivery periods of the movement requirements, that is $P_o \subset P$.

The set L of all combinations of routes, opportune delivery periods, and cargo types for movement requirements is:

$$L = \{ R \times P_o \times H \}.$$

The set $\tilde{L} \subset L$ of allowed combinations depends on the existence of a movement requirement involving a particular route, opportune delivery period, and cargo type combination. The set S of all combinations of aircraft types available in a given mission period is:

$$S = \{ A \times P_m \}.$$

The set $\tilde{S} \subset S$ of allowed combinations depends on whether there are any aircraft available during a given mission period.

The set Q of all possible combinations for transporting movement requirements on different aircraft types during mission periods is:

$$Q = \{ (A \times P_m \in \tilde{S}) \times (R \times P_o \times H \in \tilde{L}) \}.$$

The set $\tilde{Q} \subset Q$ is the allowed combinations of A , R , P_m , and H such that nonzero route capacity exists and a compatible aircraft and cargo combination exist. Additionally, the mission period in P_m must fall within the interval containing the early deviation $\underline{d}_{r \tau h}$ and late deviation $\overline{d}_{r \tau h}$ from the opportune delivery period in P_o and must also be later than the available to load period $ALP_{r \tau h}$ for a particular movement requirement. This set ensures the combination of primary indices occurs only when there is a movement requirement and the aircraft and route capacity exists to deliver the requirement within mission periods which fall inside the interval of permissible periods. The set $\tilde{Q}_{at}$ of all movement requirements using a particular aircraft type a and mission period t is:

$$\tilde{Q}_{at} = \{ R \times P_o \times H \mid R \times P_o \times H \times \{(a,t)\} \in \tilde{Q} \}.$$

The set $\tilde{Q}_{r\tau h}$ of all aircraft types available during the mission periods P_m with a particular route r, opportune delivery period τ, and cargo type h is:

$$\tilde{Q}_{r\tau h} = \{ A \times P_m \mid A \times P_m \times \{(r, \tau, h)\} \in \tilde{Q} \}.$$

In addition to the conditions mentioned above, $\tilde{L}$, $\tilde{S}$, and $\tilde{Q}$ may contain other implicit constraints which are specific to a given problem. The model is formulated as a linear program:

$$\text{Minimize} \quad \sum_{(a,r,t,\tau,h) \in \tilde{Q}} C^X_{art\tau h} X_{art\tau h} + \sum_{(r,\tau,h) \in \tilde{L}} C^N_{r\tau h} N_{r\tau h}$$

Subject To

$$\sum_{(r,\tau,h) \in \tilde{Q}_{at}} X_{art\tau h} / RCAP_{art} \leq ASUP_{at} \qquad (a,t) \in \tilde{S} \qquad (1.1)$$

$$\sum_{(a,t) \in \tilde{Q}_{r\tau h}} X_{art\tau h} + N_{r\tau h} = MREQ_{r\tau h} \qquad (r,\tau,h) \in \tilde{L} \qquad (1.2)$$

linear side constraints involving $X_{art\tau h}$ (1.3)

$$0 \leq X_{art\tau h} \qquad (1.4)$$

$$0 \leq N_{r\tau h} \qquad (1.5)$$

Equation (1.1) ensures that all movement requirements are satisfied by actual shipments or infeasibility variables. Equation (1.2) ensures the number of aircraft deployed for missions does not exceed the available pool of aircraft. Equation (1.3) represents linear side constraints such as those associated with port throughput restrictions. An example of such constraints for ports of embarkation is:

$$\sum_{\substack{(a,\tau,h) \\ (r=(i,j) \mid (i,j) \in \{ i \} \times J)}} X_{art\tau h} \leq PCAP_{it} \; \forall \; (i,t). \tag{1.3'}$$

A similar set of constraints could be constructed for the debarkation ports. Equations (1.4) and (1.5) are standard nonnegativity constraints.

The first two constraint sets, equations (1.1) and (1.2) form the generalized transportation network. It is noted that port capacity constraints (1.3′) could be included as generalized network constraints by expanding the transportation network to a transhipment network. This is not pursued here since not all types of side constraints can be handled in this manner. A simple three period depiction for the bipartite graph of the generalized transportation network is shown in Figure 1. Nodes are divided into those related to the supply of aircraft of a particular type available during a mission period ($\tilde{S}$) and those related to the movement requirements ($\tilde{L}$) . The arcs, represented by the variables, are directed from $\tilde{S}$ to $\tilde{L}$. Direct period delivery arcs are shown with solid lines while the cross period delivery arcs are shown with dashed lines.

The cost scheme for the variables establishes a set of hierarchical penalties based on various aspects of the deployment problem. The primary tier of the cost scheme involves the timeliness of delivery for the three different classes of assignment. There is no penalty for the direct delivery of a movement requirement. Cross period delivery penalties of $PPEN_{t\tau}$ are assessed dependent upon whether a delivery is late or early and upon the amount of the deviation from the opportune delivery period. The penalty structure for cross period variables is an increasing function of the earliness and lateness of movement requirement delivery. This function imposes a precedence pattern on the model which implies a delivery of one day early is better than one day late which is better than two days early and so forth. The following relation illustrates this priciple:

$$0 = PPEN_{\tau,\tau} < PPEN_{\tau-1,\tau} < PPEN_{\tau+1,\tau} < PPEN_{\tau-2,\tau} \quad \ldots$$

This pattern progresses throughout the entire interval of permissible periods for delivery.

Two other penalty structures subordinate to the first are imposed, which allow further refinements of the objectives for assigning aircraft to movement requirements. These two additional penalty structures form the second and third tiers of the penalty scheme and are based on the relative priority to be used for assigning cargo and aircraft types to missions. The second tier of the penalty scheme assesses penalties of $HPEN_h$

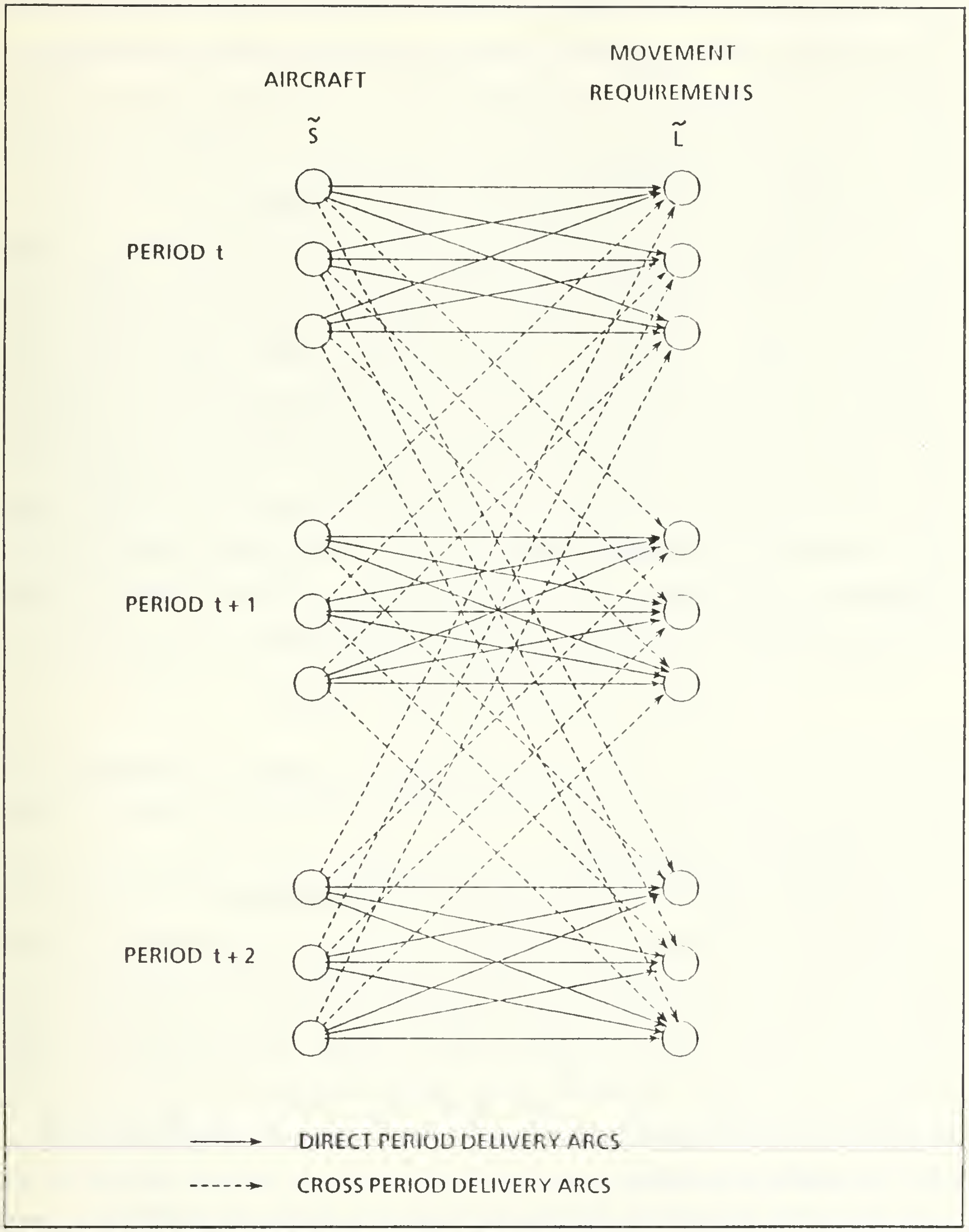

Figure 1. Generalized Transportation Network

based on the assignment priorities for cargo classes involved with cross period deliveries. This penalty tier is used to prioritize cargo in the event of an early or late delivery. A tertiary tier of penalties $APEN_a$, assesses penalties for using particular types of aircraft for any type of feasible delivery. This penalty tier is used because certain types of aircraft may be considered to be more suitable to the military operation being considered, while other aircraft may be more valuable as a reserve component.

The hierarchical penalty structure gives a precedence relation for assessing the importance of variables when higher level penalties are equal. It is noted that each of the three penalty tiers are defined over index sets which are independent of the index sets of other penalty tiers. Therefore, the individual penalties within a penalty tier are specifically related to its index set. Additionally, all subordinate penalties have a sufficiently low value so that they do not influence the penalty structure of higher tier penalties, that is, lower tier penalties are smaller than the smallest difference of penalty magnitude between any two penalties associated with a higher tier. This creates a cost scheme which allows the additive combination of priorities without entangling the various objectives represented by the precedence relations of the different tiers. The costs associated with particular variables is derived form combining penalties as follows:

$$C^X_{artzh} = PPEN_{tz} + HPEN_h + APEN_a .$$

Additionally, the cost for infeasible deliveries of C^X_{rth} is set larger than the maximum of the feasible variable delivery costs in order to ensure that infeasibility variables are used only if feasible delivery is impossible.

C. MOVEMENT REQUIREMENT REALLOCATION MODEL

The Generalized Transportation Formulation may have a large number of variables. A large problem for an 80286-based personal computer might involve twelve time periods, four aircraft types, two cargo types, and nine routes. There is the possibility that over 10,000 variables could be generated which would most likely make the problem intractable to solve using a general linear programming package. For this reason, a transformation of the Generalized Transportation Formulation is proposed in order to reduce the number of variables.

In the original formulation, variables are required to track each possible movement requirement that could be assigned to various missions. An alternative method for tracking assignments is to create separate variable types for the quantity of cargo amongst all opportune delivery periods assigned to each mission and for the quantity of movement requirements which are delivered in periods other than the opportune delivery period regardless of aircraft type. With this alternative method of variable generation, the direct assignment of cross period deliveries of aircraft missions to movement requirements is avoided. This limits the size of the problem mentioned above to under 3700 variables. This new model is equivalent to the Generalized Transportation Formulation with following exception. In the Generalized Transportation Formulation, the type of aircraft was determined for the cargo of movement requirements which was delivered early and late as well as that which was delivered on time. In the new model, the cargo which is delivered early or late will be reallocated to cargo of some other period's movement requirement. Therefore, this late or early cargo is not specifically tracked as a separate delivery on a particular aircraft type, but rather as a part of a different movement requirement. The information lost is the aircraft type which the early or late cargo arrived on. This information is usually irrelevant since it does not matter which aircraft the late or early cargo arrived on, but rather which cargo arrived early or late and how early or late it was. The transformation is implemented using an aggregation of shipments across opportune delivery periods for movement requirements using a particular route and cargo type delivered during a specific mission period as follows:

$$X'_{ar\tau h} = \sum_{t} X_{art\tau h} \, .$$

Additionally, aircraft types are aggregated for each mission period involved with a movement requirement as follows:

$$Y_{rt\tau h} = \sum_{a} X_{art\tau h} \qquad (t \neq \tau).$$

The transformation of $X_{art\tau h}$ into these two new variable types is derived by separating the possible variables into direct period deliveries and cross period deliveries. The direct period deliveries can further be separated into a positive term involving all possible cargo shipped on a particular mission and a negative term involving all early or late cargo shipped on the mission. Algebraically, the derivation of the transformation for a fixed (r, τ, h) is:

$$
\begin{aligned}
\sum_{a}\sum_{\iota} X_{ar\iota\tau h} &= \sum_{a}\sum_{\substack{\iota \\ \iota=\tau}} X_{ar\iota\tau h} + \sum_{a}\sum_{\substack{\iota \\ \iota\neq\tau}} X_{ar\iota\tau h} \\
&= \sum_{a}\left\{\sum_{\iota} X_{ar\tau\iota h} - \sum_{\substack{\iota \\ \iota\neq\tau}} X_{ar\tau\iota h}\right\} + \sum_{a}\sum_{\substack{\iota \\ \iota\neq\tau}} X_{ar\iota\tau h} \\
&= \sum_{a}\sum_{\iota} X_{ar\tau\iota h} - \sum_{a}\sum_{\substack{\iota \\ \iota\neq\tau}} X_{ar\tau\iota h} + \sum_{a}\sum_{\substack{\iota \\ \iota\neq\tau}} X_{ar\iota\tau h} \\
&= \sum_{a} X'_{ar\tau h} - \sum_{\substack{\iota \\ \iota\neq\tau}} Y_{r\tau\iota h} + \sum_{\substack{\iota \\ \iota\neq\tau}} Y_{r\iota\tau h}
\end{aligned}
$$

The final right hand side of this formula has three terms. The first term represents the aggregate movement requirements which are transported on a mission using aircraft type a in period τ , henceforth to be called the **mission delivery variable**. The second term represents the amount from each of the movement requirements not having an opportune delivery period of τ , which are transported in mission period τ . This is a reallocation of movement requirements from periods other than τ by regrouping them with a movement requirement which has opportune delivery period τ . The third term represents the reallocation from the movement requirement with an opportune delivery period of τ by regrouping them with movement requirements having opportune delivery periods other than τ . The variables associated with these last two terms will be called the **reallocation variables**.

When the infeasibility variable $N_{r\tau h}$ and the movement requirement $MREQ_{r\tau h}$ are included with this transformation, a restructured version of equation (1.1) of the Generalized Transportation Formulation is obtained. A conservation of flow relation exists for a specific movement requirement node involving the set of indices (r, τ, h) . Similarly, $X'_{ar\tau h}$ is substituted into the remaining equations to give commensurate transformed versions of the original equations. The new formulation is called the **Movement Requirement Reallocation Model (MRR)** The primary index sets for the model are:

$i \in I$	embarkation ports.
$j \in J$	debarkation ports.

$r \in R$ routes.

$a \in A$ aircraft types.

$h \in H$ cargo types.

$t, \tau \in P$ periods of time horizon.

The data for this model uses the same physical units convention as the generalized transportation model. The given data for the problem is:

$MREQ_{r\tau h}$ movement requirement of cargo type h in stons to be shipped over route r with opportune delivery period τ .

$ALP_{r\tau h}$ available-to-load period for a movement requirement .

$\underline{d}_{r\tau h}, \bar{d}_{r\tau h}$ number of periods which a movement requirement may be delivered early or late from its opportune delivery period.

$ASUP_{at}$ number of aircraft of type a available in mission period t .

$RCAP_{art}$ cargo capacity in stons/aircraft for aircraft type a over route r during mission period t .

$PCAP_{it}$ port throughput capacity for movement requirements involving cargo type h in stons for port i during mission period t .

$C^{X'}_{arth}$ cost per ston for shipping cargo type h with opportune delivery period τ over route r using aircraft type a during mission period t .

$C^{Y}_{r\tau h}$ cost per ston of reallocating a mission requirement of cargo type h over route r from a opportune delivery period of τ to one of t .

$C^{N}_{r\tau h}$ cost per ton for not delivering cargo within a movement requirement having cargo type h with opportune delivery period τ required over route r .

The variables for the problem are:

X'_{arth} stons of cargo type h shipped over route r using aircraft type a during mission period t .

$Y_{r\tau h}$ stons of a mission requirement with cargo type h over route r with opportune delivery period τ which is reallocated to mission period t .

$N_{r\tau h}$ undelivered stons of a movement requirement having cargo type h with opportune delivery period τ required over route r .

In addition to the primary index sets, the following derived index sets are necessary. The sets P_m , P_o , and $\tilde{S}$ are defined the same as in the General Transportation Formulation. The set L' of the cartesian product of routes, periods, and cargo types for movement requirements is:

$$L' = \{ R \times P_o \times H \}.$$

The set $\tilde{L}' \subset L'$ contains the allowed combinations of route, period and cargo indices. At least one of two conditions must occur for an index combination to be a member of $\tilde{L}'$. Either a movement requirement involving a route, the opportune delivery period, and the cargo type exists, or there are aircraft types available to deliver movement requirements during the opportune delivery period. The last condition does not require a nonzero movement requirement to exist since the model allows the reallocation of compatible movement requirements from other opportune delivery periods.

The set V of all possible combinations of indices for transporting movement requirements on different aircraft types during the possible mission periods is:

$$V = \{ A \times R \times P_m \times H \}.$$

The set $\tilde{V} \subset V$ restricts this set by allowing only combinations $a,\ r,\ t \in P,\ h$ such that a nonzero route capacity exists and there are aircraft available which are compatible with the cargo type. The set W of all possible combinations of mission periods with movement requirements is:

$$W = \{ R \times P_m \times P_o \times H \}.$$

The set $\tilde{W} \subset W$ of allowed combinations contains only indices which have nonzero route capacity for at least one of the available aircraft types of a specified movement requirement. The aircraft type must be compatible with the particular cargo type of H . Additionally, the mission period in P_m must fall within the interval containing the early deviation $\underline{d}_{r\tau h}$ and late deviation $\overline{d}_{r\tau h}$ from the opportune delivery period in P_o and must be later than the available loading period $ALP_{r\tau h}$. This set ensures that the combination of primary indices occurs only if a movement requirement exists and there are compatible aircraft and route capacity available which could deliver it using mission periods which fall within the interval of permissible periods.

The set $\tilde{V}_{r\tau h}$ of all aircraft types which can be used for a particular movement requirement is:

$$\tilde{V}_{r\tau h} = \{ A \mid A \times \{(r, \tau, h)\} \in \tilde{V} \}.$$

The set $\tilde{V}_{at}$ of all routes and cargo types which can be used with a particular aircraft type and mission period combination is:

$$\tilde{V}_{at} = \{ R \times H \mid R \times H \times \{(a,t)\} \in \tilde{V} \}.$$

The set $\tilde{W}_{r\tau h}$ of all mission periods for a particular movement requirement is:

$$\tilde{W}_{r\tau h} = \{ P_m \mid (P_m \times \{(r,\tau,h)\} \in \tilde{W}),\ \{\tau\} \in P_o \}.$$

The set $\tilde{W}'_{r\tau h}$ of all opportune delivery periods for a particular route, mission period, and cargo type combination is:

$$\tilde{W}'_{r\tau h} = \{ P_o \mid (P_o \times \{(r,\tau,h)\} \in \tilde{W}),\ \{\tau\} \in P_m \}.$$

In addition to the conditions mentioned above above, $\tilde{L}'$, $\tilde{S}$, $\tilde{V}$, and $\tilde{W}$ may contain other implicit constraints which are specific to a given problem. The variables within the problem are restricted by sets of allowed combinations of primary indices. The variables X'_{arth} are defined over the derived set $\tilde{V}$, while the variables $Y_{rt\tau h}$ are defined over the derived set $\tilde{W}$. The problem is formulated as a linear program:

$$\text{Minimize} \quad \sum_{(a,r,t,h)\in \tilde{V}} C^{X'}_{arth} X'_{arth} + \sum_{(r,t,\tau,h)\in \tilde{W}} C^{Y}_{rt\tau h} Y_{rt\tau h} + \sum_{(r,\tau,h)\in \tilde{L}'} C^{N}_{r\tau h} N_{r\tau h}$$

Subject To

$$\sum_{(r,h)\in \tilde{V}_{at}} X'_{arth}/RCAP_{art} \le ASUP_{at} \qquad (a,t) \in \tilde{S} \quad (2.1)$$

$$\sum_{a\in \tilde{V}_{r\tau h}} X'_{ar\tau h} - \sum_{\substack{t\in \tilde{W}'_{r\tau h} \\ t\neq\tau}} Y_{r\tau th} + \sum_{\substack{t\in \tilde{W}_{r\tau h} \\ t\neq\tau}} Y_{rt\tau h} + N_{r\tau h} = MREQ_{r\tau h} \qquad (r,\tau,h) \in \tilde{L}' \quad (2.2)$$

linear side constraints involving X'_{arth} and $Y_{rt\tau h}$ (2.3)

$$0 \le X'_{arth} \qquad (2.4)$$

$$0 \leq Y_{rt\tau h} \tag{2.5}$$

$$0 \leq X_{r\tau h} \tag{2.6}$$

The interpretation of the constraints in the Movement Requirement Reallocation Model is similar to that of the constraints in the Generalized Transportation Formulation. Constraint (2.1) ensures all movement requirements are satisfied by mission delivery variables, using reallocation variables to transfer movement requirements into or out of the node which is represented by the constraint. Constraint (2.2) ensures the aircraft pool is not exhausted. Figure 2 shows a simple depiction of the network relation obtainable from these two constraint sets. The mission delivery arcs are shown with solid lines while the reallocation arcs are shown with dashed lines. Equation (2.3) represents the linear configuration constraints while equations (2.4), (2.5), and (2.6) are nonnegativity constraints.

Since the network involved with the transformed model is no longer a bipartite graph directed from $\tilde{S}$ to $\tilde{L}'$, there is the distinct possibility of **illegitimate flows** to exist. An illegitimate flow exists when nonzero reallocations occur on a directed path of reallocation arcs connecting three or more mission requirement nodes. In the event the ending node is the same as the originating node for the path, a cycle is created. A directed path implies that a node on the path will have both an arc importing flow into it and another arc exporting flow from it. If an illegitimate flow exists in the model, then a movement requirement is shipped with a lower total cost by using successive reallocations across intermediate time periods. This avoids incurring the larger cost of reallocating a movement requirement directly from the opportune delivery period to the mission period. Since the resulting shipment by either reallocation process is the same, the use of the intermediate transfers violates the intended relative cost structure of the model.

To prevent the occurrence of illegitimate flows in the model, a restriction on the costs for the reallocation variables and the infeasibility variables of the objective function is required. Letting the subset $\{ k,l,m \} \subset P_o$ represent any three time periods of the set of opportune delivery periods with both r and h being fixed, the following restriction will be imposed on the costs involving a movement requirement reallocation between periods k and m:

$$C^{X}_{rlh} < C^{Y}_{rklh} + C^{Y}_{rlmh} \ . \tag{1}$$

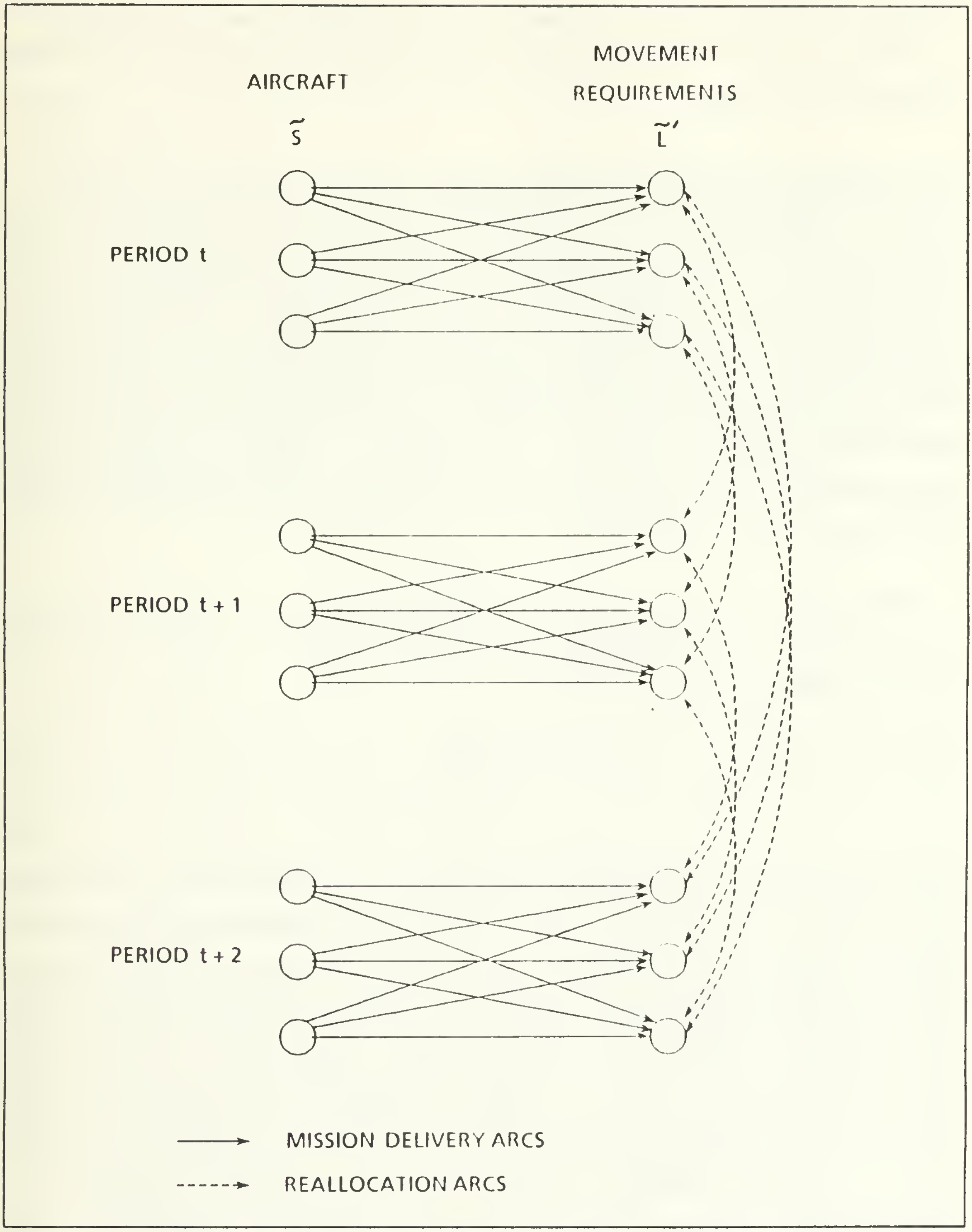

Figure 2. Movement Requirement Reallocation Network

Lemma: With regard to the Movement Requirement Reallocation Model, condition (1) is sufficient to prevent the simultaneous importation and exportation of flow on reallocation arcs at a $\tilde{L}'$ node.

Proof: Let node l be the $\tilde{L}'$ node with both route r and cargo type h fixed and with the opportune delivery period of l. Let this node contain a directed path between nodes k and m. Condition (1) states the cost for using the infeasibility arc associated with node l is less than the total cost of the two directed arcs connecting it with nodes k and m. Since the infeasibility arc associated with node l is unbounded and the Movement Reallocation Model is a minimization problem, the infeasibility arc will always be used prior to the use of any combination of two or more reallocation arcs through the directed path containing the node l. Therefore, the simultaneous importation and exportation of flow on reallocation arcs can not occur in the Movement Requirement Reallocation Model. QED.

While condition (1) is sufficient to prevent illegitimate flows in the transformed model, additional conditions are required to ensure the use of variables in the models are valid. The additional conditions are:

$$C^{Y}_{rkmh} < C^{X}_{rkh} \tag{2}$$

$$C^{X'}_{arkh} < C^{Y}_{rklh} + C^{X'}_{arlh} \ . \tag{3}$$

Condition (2) states the cost of using the reallocation variable for node k must be less than the cost of its infeasibility variable. This ensures reallocations of movement requirements occur before a movement requirement is determined to be undeliverable. It is noted that when restrictions (1) and (2) are assembled together, a triangle inequality exists separated by the cost associated with the infeasibility variable. However, the triangle inequality is not sufficient by itself to prevent illegitimate flows since a directed path could exist between nodes k, l, m while no variable existed linking nodes k and m directly. The addition of the infeasibility cost to the restriction prevents the simultaneous importation and exportation of flow for this type of directed path which would otherwise satisfy the triangle inequality. Restriction (3) is implied by the definition of the opportune delivery period. For the definition to be meaningful, movement requirements must be satisfied within missions periods which are the same as the opportune delivery period prior to the reallocation of the requirement to a node with a different opportune delivery period. Consequently, the cost of satisfying the requirement in the same mission period as the opportune delivery period must be less than the cost of reallocating it to a different opportune delivery period and the associated cost of performing the mission during that period.

The cost scheme in the General Transportation Formulation is convertible to an equivalent structure for the Movement Requirement Reallocation Model if cost restriction (1) is satisfied for both models. It is assumed restrictions (2) and (3) already exist in a form useable to the General Transportation Formulation. Because the mission delivery costs in the models are independent of the reallocation costs, the usage penalty for aircraft are exclusively assessed on the mission delivery variables since they represent the actual shipment of cargo by aircraft. Both the timeliness of delivery penalties and the cargo assignment penalties represent costs exclusively associated with the reallocation variables. Therefore, the transformed cost relations are:

$$C^{X'}_{arth} = APEN_a$$

$$C^{Y}_{rt\tau h} = PPEN_{t\tau} + HPEN_h.$$

When comparing the two models, it is important to note the distinct difference between the sets $\tilde{L}$ of the Generalized Transportation formulation and $\tilde{L}'$ of the Movement Requirement Reallocation Model. Both are used to restrict the first constraint set of their respective models. There will typically be many more constraints in the constraint set of equation (2.1) of the Movement Requirement Reallocation Model. This occurs because the index set $\tilde{L}'$ does not eliminate the node (r, τ, h) when a movement requirement does not exist for those indices. This null movement requirement node must still exist since movement requirements from other opportune delivery periods may be reallocated to it. This presents a tradeoff between the two models involving the reduction of variables versus the reduction of constraints. It is expected there will be a cross-over point for the solvability of instances of problems dependent upon the density of the number of nonzero movement requirements in relation to the total possible number of movement requirements given the size of the primary index sets. The General Transportation Formulation should perform better for low density instances of problems. The Movement Requirement Reallocation Model will improve its performance in comparison with the General Transportation Model.

D. INDEPENDENT TIME-WINDOW SEPARATION ALGORITHM

For many operation plans involving the aircraft deployment problem, the size of an instance of the model may be intractable or have an unacceptably long solution time. This section of the thesis explores a specialized solution technique which can be used to solve a certain subset of these difficult problems. The subset of problems involves those cases where either the early deliver or the late delivery of cargo is unacceptable. When the model is restricted to allowing only early or only late deliveries of cargo, a simple algorithm can be used to find independent subproblems within the entire problem.

Assuming a problem under consideration has no gross inadequacies in the available supply of aircraft with regard to the total amount of cargo to be shipped, it is anticipated that independent subgroupings of periods may exist. These subgroupings or **time-windows** form a subproblem which is independent of the remainder of the model if the movement requirements within the time-window can be satisfied entirely using the aircraft available in that same time-window. It is usually difficult to tell where these independent time-windows would occur for a particular instance of the problem without solving the entire instance. However, it is possible to iteratively define and solve subproblems of independent time-windows when either earliness or lateness is not allowed.

Consider an instance of the deployment problem where early deliveries are disallowed. A time-window is independent of future periods if no late deliveries occur outside of this time-window. This happens in general because the cost structure of the deployment models enforce the delivery of movement requirements as close to the opportune delivery period as possible. Since movement requirements can not be delivered in periods earlier than the time-window and all deliveries to time periods later the time-window have higher costs, then all deliveries are made within the time-window if there are sufficient assets available.

An iterative algorithm can be used to solve the entire problem which makes use of this property. The algorithm starts by initially solving a subproblem containing a time-window with only the first period of the time horizon. A time-window of length n is $\omega = \{ t,\ t+1,\ t+2,\ \ldots,\ t+n-1\}$, where $t+k \in P$ and $k = 0,\ 1, \ldots, n-1$. If all deliveries are made within ω , the subproblem is labeled as independent, the solution for the subproblem is saved, and ω is revised to include only the next period of the time horizon. If not, the solution is disregarded and the next period of the time horizon is added to the existing period in ω The next subproblem is solved based on the revised ω . This process of solving the subproblem and then updating ω continues until the last period of the time horizon has been solved. Thus, a forward directed algorithm is obtained which uses an ascending ordering of the periods (earliest to latest) to solve the entire problem by progressively solving subproblems over ω . A similar procedure is possible when late deliveries are disallowed which iteratively solves subproblems based on a descending progression (latest to earliest) through the periods of the time-horizon. A simple algorithm outlining the general procedure is shown in Figure 3.

This algorithm is particularly well suited to instances of the deployment problem where the demand for movement requirements fluctuates substantially from period to period or is unimodal in nature with respect to time. As the size of the problems attempted increases, the benefit in solving the smaller independent time-windows of those problems increases. Two other important benefits of the algorithm are its capability to model extended time horizons for certain instances, and its ability to be restarted from advanced positions in the time horizon. The worst case instance for this algorithm would have deliveries outside of the opportune delivery periods for all periods in the time-horizon. In this case, the algorithm will ultimately solve the entire problem, being no worse off for using the algorithm than having pointlessly expended effort solving the smaller subproblems. If the problem is still intractable after using this algorithm, there is still valuable partial information which was obtained from iterating through the various time-windows of the algorithm. Even if no independent time-windows were found, bottlenecks in the problem would most likely be identifiable from the partial information or an advanced starting position found for continuing the algorithm following time-windows which are nearly independent. Another use for the algorithm is as a tool to determine bounds on instances of problems where both early and late deliveries are allowed. The procedure would be to solve the problem twice, once where earliness was disallowed and once where lateness was disallowed. Since either condition is a restriction of the original problem, either offers a bound on the unrestricted problem. By studying both solutions, valuable information for the total solution can be obtained.

E. INTEGER ROUNDING MODEL

In order to make the solutions from the previous models compatible with the existing post-solution processor in the DAP, an integer rounding method is proposed. From the solution of any of the previous models, the amount of each movement requirement to be transported on the various aircraft types over a specific route and mission period are determined. The number of aircraft required for each mission can be obtained from this solution using either of the following equations:

```
ALGORITHM: Independent Time-Window Separator

INPUT:    Military Aircraft Deployment Problem formulated
          using a Generalized Transportation Formulation
          or Movement Requirement Reallocation Model.
          Periods P will be preordered from earliest to latest
          (ascending order).
OUTPUT:   List of aircraft mission assignments to
          movement requirement.

          { Initialize:
            IF (no lateness allowed)
              reorder P in descending order
            ω = time-window of the first period of the ordered P
            WHILE ( ω ∈ P)
            { WHILE ( MREQ(ω) = 0)  ω = next period
              IF( ω ∈ P)
              { SOLVE the subproblem over ω
                D(ω) = number of deliveries outside of ω
                IF ( D(ω) > 0)
                  { ω = ω + next period }
                ELSE
                  { ω = next period }
              }
            }
          }
          OUTPUT
```

Figure 3. Independent Time Window Separation Algorithm

$$AREQ_{art} = \sum_{(\tau,h)} X_{art\tau h} \qquad \text{Generalized Transportation Formulation}$$

$$AREQ_{art} = \sum_{h} X'_{arth} \qquad \text{Movement Requirement Reallocation Model}$$

where the variables on the right-hand side are at their optimal values. The value will often be fractional for $AREQ_{art}$.

The rounding model of this study provides one method to round the continuous solution. This model attempts to induce integer solutions by rounding up fractional values of $AREQ_{art}$ provided there are unused aircraft available of the same aircraft type as the variable being rounded. This avoids leaving cargo undelivered when there are unassigned aircraft which could deliver it. The underlying principle is to sort the fractional part of the continuous solution by magnitude from highest to lowest for each aircraft type during a mission period. The fractional parts are then rounded up to the value of one, maintaining the ordering of the sort, until supplies for an aircraft type during a mission period are exhausted. Any fractional part of a value which cannot be rounded up because of aircraft supply restrictions is omitted (implicitly truncated to zero) from the solution. This principle is programmable using a variety of methods. The **Integer Rounding Model** which follows applies this principle in a linear programming environment. The primary index sets of the Integer Rounding Model are:

A	aircraft types.
R	routes.
P_m	period of aircraft missions.
H	cargo types.

The given data for the problem is:

$ASUP_{at}$	number of aircraft type a available to perform missions during period t .
$AREQ_{art}$	continuous solution from previous model expressed as the number of aircraft used.
E_{art}	fractional part of $AREQ_{art}$

The variables for the problem are:

Z_{art}	indicator for rounding up a fractional aircraft of type a used on route r for a mission during period t.

In addition to the primary sets the following derived sets are necessary. The set $\tilde{S}$ of all possible combinations of aircraft and mission periods such that aircraft are available is the same as described in the previous models. The set f of the aircraft types, routes, and mission periods such that fractional aircraft solutions exists is:

$$f = \{ A \times R \times P_m : E_{art} \neq 0 \} .$$

The problem is formulated as a linear program:

$$\text{Maximize} \quad \sum_{(a,r,t)\in f} E_{art} Z_{art}$$

Subject To

$$\sum_{r\in R} Z_{art} \leq ASUP_{at} - \sum_{r\in R} (AREQ_{art} - E_{art}) \qquad (a,t) \in \tilde{S} \quad (3.1)$$

$$Z_{art} \in \{0,1\} \qquad (3.2)$$

The objective function maximizes the total of the fractional parts of the continuous aircraft solution which are rounded up using the binary indicator variable Z_{art}. Constraint (3.1) ensures the indicator variable Z_{art} does not exceed the remaining supply of aircraft. The remaining supply of aircraft is shown in the right hand side of the equation as the total supply of available aircraft less the integer portion of the continuous aircraft solution previously calculated. Constraint (3.2) is the zero-one restriction on Z_{art}.

The Integer Rounding Model, when solved as a continuous linear program replacing equation (3.2) with $0 \leq Z_{art} \leq 1$, will always generate integer solutions. To understand this from a mathematical programming perspective, the nature of the constraints of equation (3.1) must be explored. Letting U_{art} be the unused aircraft represented by the right hand side of equation (3.1), consider the matrix form of the relaxed integer model:

$$\max \ \{ \ \mathbf{EZ}: \ \ \mathbf{AZ} \leq \mathbf{U}, \quad 0 \leq \mathbf{Z} \leq 1 \ \}$$

where **A** is the technical coefficient matrix. Each of the constraints generated from equation (3.1) forms a set of ones for each row of the **A** matrix. Since the indices of a and t are fixed for each equation, mutually exclusive sets of ones are generated for each constraint across the routes. Therefore, there will only be a single one in any column of **A**, the rest of its elements being zero. Each of the rows of the **A** matrix are part of a set of generalized upper bound (GUB) constraints. Additionally, a simple upper bound of one is applied to each variable. The proof of integrality follows directly from the unimodularity of the **A** matrix and the integrality of both the variable bounds and **U**, which implies any extreme point is integer [Ref. 13].

At this point, it must be emphasized that this model does not necessarily provide an optimal conversion of the original deployment model solutions to integer. It is an approximation. However, the approximation should be close to the optimal solution when nontrivial numbers of aircraft are required for most routes during a military operation. Notice this condition is the same as that previously mentioned for the deployment models. Consequently, a reasonably exact measure using a rounded solution should be obtained when an instance for one of the deployment model satisfies this condition.

III. COMPUTATIONAL RESULTS

This chapter describes the results of testing representative data on the models and the algorithm previously described. The data for the tests were obtained from USTRANSCOM. The particular data set is similar to what would be expected to be solved on the DAP. The models were tested using this data on a personal computer with similar characteristics as those which the DAP is currently implemented on. The computer used was an IBM Personal System/2 Model 50 with 640k of addressable memory and a 10 Mhz 80287 math coprocessor.

The data set involves four aircraft types, nine routes, twelve time periods, and two cargo types. The nine routes are spread over three embarkation ports on the east coast of the United States and three debarkation ports in Europe. The number of aircraft available during any period ranges from 186 to 354. The test problem requires the delivery of 83 different movement requirements involving 219,693 short tons of freight. A crude approximation for the number of aircraft trips required for this problem yields over 4000 round trips between the different ports of embarkation and debarkation. This large number makes the use of a continuous variable deployment model appropriate. For this particular data set, the maximum amount of lateness for any movement requirement is three days, while the maximum amount of earliness is only restricted to periods within the time-horizon.

Considering the number of index combinations for routes, time periods, and cargo types, there are 216 different possible movement requirements which could occur for this problem. The 83 movement requirements which actually exist out of the possible 216 were considered to be representative of the relative density of the actual movement requirements to the total possible movement requirements for a problem. To examine the performance of the General Transportation Formulation (GTF) versus the Movement Requirement Reallocation model (MRR), the original set of movement requirements (data set ORIG) was expanded or reduced to yield a higher and lower density test instance. The higher density instance (data set HIGH) contained 135 movement requirements involving 225,103 short tons of freight. The lower density instance (data set LOW) contained 33 movement requirements involving 106,906 short tons of freight. All other data elements remained as in the original data set. In order to test the Independent Time-Window Separation Algorithm (ITWSA), a second comparison run was performed with lateness disallowed in the model. The ITWSA was applied to the MRR model. A test without the separation algorithm was also run for comparison.

The solution characteristics for the linear programs of the various tests are shown in Table 1, and the total GAMS execution times and the linear program solver times are shown in Table 2 and Table 3. Test type number one is for test runs where lateness was allowed in the instance and test type number two is for test runs where lateness was disallowed. The type of solution technique is indicated by the name of the model.

Table 1. LINEAR PROGRAM SOLUTION CHARACTERISTICS

MODEL DATA SET	LOW	ORIG	HIGH
GTF-1			
equations	144	196	
variables	742	1613	
nonzero coefficients	3607	7812	----
MRR-1			
equations	337	337	337
variables	760	1077	1463
nonzero coefficients	3325	4226	5332
MRR-2			
equations	337	337	337
variables	665	837	1078
nonzero coefficients	3040	3506	4177
ITWSA-2			
equations	347	347	347
variables	596	662	714
nonzero coefficients	2807	2947	3791
max eqns./iteration	57	57	57
max vars./iteration	103	119	119
max nzcoef./iteration	481	521	521
NOTATION:			
GTF-1 Generalized Transportation Formulation - run 1			
MRR-1 Movement Requirement Reallocation Model - run 1			
MRR-2 Movement Requirement Reallocation Model - run 2			
ITWSA-2 Indep. Time-Window Separation Algorithm - run 2			

Table 2. TOTAL EXECUTION TIME (minutes)

MODEL DATA SET	LOW	ORIG	HIGH
GTF-1	9.92	25.12	----
MRR-1	10.62	13.32	16.42
MRR-2	10.18	11.67	13.77
ITWSA-2	13.03	13.48	14.37

Table 3. LINEAR PROGRAM SOLVE TIME (minutes)

MODEL DATA SET	LOW	ORIG	HIGH
GTF-1	5.00	17.12	----
MRR-1	5.75	7.23	9.45
MRR-2	5.13	6.22	7.50
ITWSA-2	1.57	1.73	1.87

The GTF required a consistently lower number of equations in comparison to that of the MRR model. This allowed the GTF to perform well for the lower density instance; however, as the density increased, its size and solution time increased sharply. The increase was so severe, that the GAMS program generator was unable to create the linear program for the higher density instance because of inadequate memory capacity in the computer. This sharp increase in problem size in relation to the number of movement requirements demonstrates that this model's ability to solve the deployment problem is highly dependent on the density of the movement requirements. This property is not desirable since a variety of instances of the deployment problem may need to be solved.

The MRR model had a slightly larger size for the lower density instance than that of the GTF. The growth in model size was very moderate from the lower density instance to the original and higher density instances which is in sharp contrast to the rapid growth found in the GTF. Despite the large disparity in the number of movement requirements used in these different instances, the MRR model remained stable in both size and solution time. This stability is directly related to the constant number of equations found in all instances of the problem, as well as the smaller quantity of variables and nonzero coefficients required when compared to the GTF.

The ITWSA performed quite well for this particular data set. The algorithm was able to identify eleven independent time windows in the twelve period problem. The accumulated linear program solution times were extremely low. The total execution time was stable; however, it was also larger than the nonseparated MRR model solution time. While the solution time was small, the total execution time was larger because of the additional overhead of generating all of the subproblems. This data set demonstrates the usefulness of the algorithm; however, it is not appropriate to think all data sets would perform nearly as well. The algorithm's performance is based on the ability to find independent time-windows within a problem. For problems with few or no independent time-windows, the solution time would be expected to increase substantially. Although, the maximum problem size would never be any larger than that which would be required to solve a problem without the separation algorithm.

The Integer Rounding Model was used to transform the continuous solutions of previous model test runs to integer values. The rounding model performed well giving deployment plans which were consistent with the objectives indicated in the the data set. The quality results are directly related to the large number of aircraft required across the different routes and time periods. Additionally, it is noted that the time required to run the Integer Rounding Model was modest.

The utility or supportability of the operation plan from the perspective of air transportation is indicated by the number of aircraft required and the amount of early, late, and undelivered cargo. For this particular data set, 14,247 stons of cargo were delivered one period early and 2,284 stons of cargo were delivered one period late. All of the remaining 203,162 stons of cargo were delivered during their opportune delivery periods. If the periods are considered to be of one day duration, then the shipment of all movement requirements used 1455 aircraft-days for the continuous variable solution and 1477 aircraft-days for the rounded solution. The overall utility of the operation plan would depend on these numbers being acceptable for the military actions being considered.

For comparison, approximations for this deployment problem were calculated using the ITWSA. When lateness was disallowed, 14247 stons of cargo were delivered one period early while 2284 stons went undelivered. When earliness was disallowed, 11195 stons of cargo were delivered one period late and 5336 stons were delivered two periods late. There was no undelivered cargo when earliness was disallowed. Bounds on the amount of early, late, and undelivered cargo are easily determined from these solutions. Comparisons of the number of aircraft-days required is not as straight-forward. As the amount of undelivered cargo increases the number of aircraft-days required decreases, but, as the amount of earliness or lateness increases, the number of aircraft-days increases or decreases depending of the size of the aircraft and the priority of usage for the different aircraft types. For the current penalty scheme, the number of aircraft-days required decreased for both the no-earliness and no-lateness approximations.

IV. SUMMARY

This thesis has presented techniques which can be applied to the aircraft deployment problem. A formulation based on the generalized transportation problem laid the foundation for exploring other methods. This formulation, in itself, was shown to be potentially unstable in size and solution time within the limitations of the solver and computer when solved as a linear program. This instability depended directly on the number of movement requirements in a given instance of the problem. Despite this, the model may still be useful with other optimization techniques such as decomposition, dynamic column generation, or network algorithms. The GAMS modelling program for this Generalized Transportation Formulation is given in Appendix A.

A transformation of the Generalized Transportation Formulation was performed by separating the original transportation variables into those which were used for satisfying movement requirements in the time they were required and those which were used to satisfy requirements which were early or late. These variables were aggregated to produce an efficient reformulation of the former model. This model was found to be quite stable and performed admirably on a personal computer. It is the model of choice when the problem is solved as a standard linear program. The GAMS modelling program for the Movement Requirement Reallocation model is presented in Appendix B.

A time window separation algorithm was proposed as an instrument to attack large problem instances. The model provides optimal solutions for problems which allow only early or only late deliveries of movement requirements. The model can also be used to find bounds on the solution for models allowing both earliness and lateness by restricting the earliness or lateness variables. The model attempts to find independent groups of time periods within a problem so that the entire horizon of time periods does not have to be solved in its entirety for any iteration of the algorithm. Rather, a collection of subproblems are solved giving a set of solutions which can be assembled into the total problem solution. The MRR model in Appendix B is equipped with the Independent Time-Window Separation Algorithm.

Lastly, an integer rounding model was proposed which allocates the fractional part of the continuous solution to the residual supply of available aircraft. This model is an effective tool to convert continuous variable solutions to integer when there are not substantial numbers of low values to be rounded. Although this model has integer variables, it can be solved as a continuous linear program with guaranteed integrality. The model provided quality conversions of the continuous solutions of the former models for the data sets tested. The GAMS modelling program for the submodule of the Integer Rounding Model is presented in Appendix C.

APPENDIX A. GAMS PROGRAM FOR THE GENERAL TRANSFORMATION FORMULATION

```
$TITLE   GENERAL TRANSFORMATION FORMULATION    VERSION (89.2.17)

$ONTEXT
        MICHAEL C. PUNTENNEY

        ACKNOWLEDGEMENT:  THIS MODEL IS BASED ON THE ORIGINAL AIR
                          LIFT ALLOCATION PROBLEM DESCRIBED IN
                          THE TRANSCOM LIFT OPTIMIZER VERSION
                          (88.6.14) BY RICHARD E. ROSENTHAL.

$OFFTEXT

$OFFUPPER  OFFSYMXREF  OFFSYMLIST


*------------------
*      DATA
*------------------

 SETS
    Pm   all periods in time horizon      /      /
    A    aircraft types                   /      /
    H    cargo types                      /      /
    I    ports                            /      /
    R    routes                           /      /;
    ALIAS(Pm,Po);



 SETS
    O(R,I)  mapping of route to embarkation port
        /        /
    D(R,I)  mapping of route to debarkation port
        /        /
    COM(A,H)  compatible aircraft cargo pairings
        /        /;


 TABLE    ASUP(Pm,A)     quantity of available aircraft ;
 TABLE    MREQ(R,Pm,H)   movement requirement ;
 TABLE    ALP(R,Pm,H)    ordinate of available to load period for Pm ;
 TABLE    RCAP(A,R,Pm)   freight capacity of aircraft types on routes ;

 PARAMETER  PCAP(I,Pm)    freight throughput limit at ports ;

 SCALARS
    MAXLTE       maximum allowable number of late delivery periods;
    MAXERL       maximum allowable number of early delivery periods;
```

```
*------------------
* DERIVED DATA
*------------------

* Revise RCAP and ASUP if zero aircraft of route capacity are available
    RCAP(A,R,Pm)$(ASUP(Pm,A) EQ 0)  =  0 ;
    ASUP(Pm,A)$(SUM(R, RCAP(A,R,Pm)) EQ 0) = 0 ;


 SET Q(A,R,Pm,Po,H) derived index set for allowed X variables;

    Q(A,R,Pm,Po,H)$COM(A,H) = YES$(( ORD(Pm) - ORD(Po) LE MAXLTE )
                            AND  ( ORD(Po) - ORD(Pm) LE MAXERL )
                            AND  ( ORD(Pm) GE ALP(R,Po,H) )
                            AND  ( MREQ(R,Po,H) GT 0 )
                            AND  ( RCAP(A,R,Pm) GT 0 ) );


*------------------
* PENALTY STRUCTURE
*------------------

 PARAMETERS
    APEN(A)         aircraft priorities
    HPEN(H)         cargo priorities
    TPEN(Pm,Po)     time of delivery penalty from Po to Pm
    NPEN            aircraft infeasibility override penalty ;

 SCALAR
    ASCALE         scaling parameter for APEN
    HSCALE         scaling parameter for HPEN
    SCALE          scaling factor for solution stability   /100.00/;


*  Default HPEN(H) assumes cargos entered in ascending priority:
    HPEN(H) = ORD(H) ;

*  Note: APEN(A) values specify order in which assets should be used,
*  i.e., highest value signifies most important to conserve.
*  Default APEN(A) assumes assets entered in descending priority.
    APEN(A) =  CARD(A) - ORD(A) + 1 ;

*  Default TPEN(Pm,Po)
    TPEN(Pm,Po) $ ( ORD(Pm) GT ORD(Po) ) =
              2 + (ORD(Pm) - ORD(Po)) / CARD(Pm) ;
    TPEN(Pm,Po) $ ( ORD(Pm) LT ORD(Po) ) =
              2 + (ORD(Po) - ORD(Pm)) / CARD(Pm) - 1 / ( 2 * CARD(Pm));
    TPEN(Pm,Po) $ ( ORD(Pm) EQ ORD(Po) ) = 0 ;

*  Determine penalty scale magnitudes ;
    HSCALE = ( 0.9 * 1/(2*CARD(Pm)) ) / CARD(H) ;
    ASCALE = 0.9 * HSCALE / CARD(A) ;

*  Scale APEN and HPEN to fit penalty hierarchy
```

```
    APEN(A) = ASCALE * APEN(A) ;
    HPEN(H) = HSCALE * HPEN(H) ;

*  Default NPEN greater than any penalty associated with Y and less
*  than twice the minimum penalty associated with Y.
    NPEN = 3.5  ;


*------------------
* PROBLEM STRUCTURE
*------------------

 VARIABLES
    X(A,R,Pm,Po,H) movement requirement assigned to aircraft mission
    N(R,Pm,H)      infeasible movement requirements
    COST           total penalty costs ;

 POSITIVE VARIABLES X,N;

 EQUATIONS
    BALANCE(R,Po,H)   maintain freight balance constraints
    ASSETMAX(A,Pm)    observe maximum available assets
    THRUPUT(I,Pm)     observe throughput limits at ports
    OBJDEF            define objective function ;


  OBJDEF ..
         SUM((A,R,Pm,Po,H)$Q(A,R,Pm,Po,H), X(A,R,Pm,Po,H)
            * (APEN(A) + TPEN(Pm,Po) + HPEN(H)$(ORD(Pm) NE ORD(Po))))
      +  SUM((R,Po,H), (NPEN + HPEN(H)) * N(R,Po,H)$MREQ(R,Po,H))
    =E=  COST ;

  BALANCE(R,Po,H)$MREQ(R,Po,H)  ..
         SUM((A,Pm)$Q(A,R,Pm,Po,H), X(A,R,Pm,Po,H))
      +  N(R,Po,H) $ MREQ(R,Po,H)
    =G=  MREQ(R,Po,H) / SCALE ;

  ASSETMAX(A,Pm) $ ASUP(Pm,A) ..
         SUM((R,Po,H)$Q(A,R,Pm,Po,H),
                 X(A,R,Pm,Po,H)/RCAP(A,R,Pm)) * SCALE
    =L=  ASUP(Pm,A) ;

  THRUPUT(I,Pm) $ ( PCAP(I,Pm) LT +INF ) ..
         SUM(R$O(R,I), SUM((A,H,Po)$Q(A,R,Pm,Po,H), X(A,R,Pm,Po,H)))
      +  SUM(R$D(R,I), SUM((A,H,Po)$Q(A,R,Pm,Po,H), X(A,R,Pm,Po,H)))
    =L=  PCAP(I,Pm) / SCALE ;


*------------------
*  MODEL / SOLVER
*------------------

 MODEL LIFTOPT /ALL/ ;
 OPTION LIMROW=0, LIMCOL=0, SOLPRINT=OFF, ITERLIM=5000;
 SOLVE LIFTOPT USING LP MINIMIZING COST ;
```

```
PARAMETER XA(R,Pm,H,A)     display aircraft mission cargo assignment
          XD(R,Pm,Po,H)    display late or early movement requirements
          PLANES(R,Pm,A)   display aircraft assignment by mission;

   XA(R,Pm,H,A) = SUM(Po, X.L(A,R,Pm,Po,H)) * SCALE ;

   XD(R,Pm,Po,H)$(ORD(Pm) NE ORD(Po)) =
                   SUM(A, X.L(A,R,Pm,Po,H)) * SCALE ;

   N.L(R,Pm,H) = N.L(R,Pm,H) * SCALE ;

   PLANES(R,Pm,A) $ RCAP(A,R,Pm)  =
      SUM( H$COM(A,H), XA(R,Pm,H,A) ) / RCAP(A,R,Pm) ;

OPTION XA:2:3:1;
OPTION XD:2:3:1;
OPTION PLANES:2:2:1;

DISPLAY XA, XD, N.L, PLANES;
```

APPENDIX B. GAMS PROGRAM FOR THE MOVEMENT REQUIREMENT REALLOCATION MODEL

USING THE INDEPENDENT TIME WINDOW SEPARATION ALGORITHM

```
$TITLE   MOVEMENT REQUIREMENT REALLOCATION TRANFORMATION (II)
*        WITH INDEPENDENT TIME WINDOW SEPARATION ALGORITHM
*        ( MAINFRAME VERSION - LOOPING SOLVE TECHNIQUE )
*        VERSION (88.7.26),   MODIFIED (89.2.17)
$ONTEXT
        MICHAEL C. PUNTENNEY

        ACKNOWLEDGEMENT:  THE MRR TRANSFORMATION IS BASED
                          ON THE ORIGINAL AIR LIFT ALLOCATION
                          PROBLEM DESCRIBED IN THE TRANSCOM
                          LIFT OPTIMIZER VERSION (88.6.14),
                          BY RICHARD E. ROSENTHAL.  THE ITWS
                          ALGORITHM IS BASED ON VERSION (87.12.23).

$OFFTEXT

$OFFUPPER  OFFSYMXREF  OFFSYMLIST


*------------------
*      DATA
*------------------

 SETS
   Pm               all periods in time horizon          /     /
   Tm(Pm)           window of periods for current solve iteration
   TITER(Pm)        iteration counter ;
   ALIAS(Pm,Po,PP,PL);
   ALIAS(Tm,To);

 SETS
   A    aircraft types                     /      /
   H    cargo types                        /      /
   I    ports                              /      /
   R    routes                             /      /;


 SETS
   O(R,I)  mapping of route to embarkation port
      /       /
   D(R,I)  mapping of route to debarkation port
      /       /
   COM(A,H)  compatible aircraft cargo pairings
       /       /;
```

```
TABLE     ASUP(Pm,A)     quantity of available aircraft ;
TABLE     MREQ(R,Pm,H)   movement requirement ;
TABLE     ALP(R,Pm,H)    ordinate of available to load period for Pm ;
TABLE     RCAP(A,R,Pm)   freight capacity of aircraft types on routes ;

PARAMETER  PCAP(I,Pm)     freight throughput limit at ports ;

SCALARS
   MAXLTE        maximum allowable number of late delivery periods;
   MAXERL        maximum allowable number of early delivery periods;


*------------------
*   DERIVED DATA
*------------------

* Revise RCAP and ASUP if zero aircraft of route capacity are available
   RCAP(A,R,Pm)$(ASUP(Pm,A) EQ 0)  =  0 ;
   ASUP(Pm,A)$(SUM(R, RCAP(A,R,Pm)) EQ 0) = 0 ;


SET W(R,Pm,Po,H)  derived index set for allowed Y variables;

   W(R,Pm,Po,H) = YES $(( ORD(Pm) NE ORD(Po) )
                   AND  ( ORD(Pm) - ORD(Po) LE MAXLTE )
                   AND  ( ORD(Po) - ORD(Pm) LE MAXERL )
                   AND  ( ORD(Pm) GE ALP(R,Po,H))
                   AND  ( MREQ(R,Po,H) GT 0 )
                   AND  ( SUM( A$COM(A,H), RCAP(A,R,Pm) ) GT 0 ) );


*------------------
* PENALTY STRUCTURE
*------------------

PARAMETERS
   APEN(A)        aircraft priorities
   HPEN(H)        cargo priorities
   TPEN(Pm,Po)    time of delivery penalty from Po to Pm
   NPEN           aircraft infeasibility override penalty ;

SCALAR
   ASCALE        scaling parameter for APEN
   HSCALE        scaling parameter for HPEN
   SCALE         scaling factor for solution stability   /100.00/
   TOLER         zero tolerance for defininig delay      /   .01/ ;

*  Default HPEN(H) assumes cargos entered in ascending priority:
   HPEN(H) = ORD(H) ;

*  Note: APEN(A) values specify order in which assets should be used,
*  i.e., highest value signifies most important to conserve.
*  Default APEN(A) assumes assets entered in descending priority.
   APEN(A) =  CARD(A) - ORD(A) + 1 ;
```

```
*  Default TPEN(Pm,Po)
    TPEN(Pm,Po) $ ( ORD(Pm) GT ORD(Po) ) =
              2 + (ORD(Pm) - ORD(Po)) / CARD(Pm) ;
    TPEN(Pm,Po) $ ( ORD(Pm) LT ORD(Po) ) =
              2 + (ORD(Po) - ORD(Pm)) / CARD(Pm) - 1 / ( 2 * CARD(Pm));

*  Determine penalty scale magnitudes ;
    HSCALE = ( 0.9 * 1/(2*CARD(Pm)) ) / CARD(H) ;
    ASCALE = 0.9 * HSCALE / CARD(A) ;

*  Scale APEN and HPEN to fit penalty hierarchy
    APEN(A) = ASCALE * APEN(A) ;
    HPEN(H) = HSCALE * HPEN(H) ;

*  Default NPEN greater than any penalty associated with Y and less
*  than twice the minimum penalty associated with Y.
    NPEN = 3.5  ;


*------------------
* PROBLEM STRUCTURE
*------------------

 VARIABLES
    X(A,R,Pm,H)    cargo assigned by aircraft mission
    Y(R,Pm,Po,H)   late or early delivry of movement requirements
    N(R,Pm,H)      infeasible movement requirements
    COST           total penalty costs ;

 POSITIVE VARIABLES X, Y, N ;

 EQUATIONS
    BALANCE(R,Pm,H)    maintain freight balance constraints
    ASSETMAX(A,Pm)     observe maximum available assets
    THRUPUT(I,Pm)      observe throughput limits at ports
    OBJDEF             define objective function ;


  OBJDEF ..
    SUM( (R,Tm,To,H), (HPEN(H)+TPEN(Tm,To)) * Y(R,Tm,To,H)$W(R,Tm,To,H))
    +    SUM( (R,Tm,H), (NPEN+HPEN(H)) * N(R,Tm,H)$MREQ(R,Tm,H) )
    +    SUM((A,R,Tm,H)$COM(A,H), APEN(A) * X(A,R,Tm,H)$RCAP(A,R,Tm) )
    =E=  COST ;

  BALANCE(R,Pm,H)$Tm(Pm)  ..
     SUM( A$COM(A,H), X(A,R,Pm,H) $ RCAP(A,R,Pm) )
   - SUM( Po $Tm(Po),  Y(R,Pm,Po,H) $W(R,Pm,Po,H) )
   + SUM( Po $Tm(Po),  Y(R,Po,Pm,H) $W(R,Po,Pm,H) )
   + N(R,Pm,H)$MREQ(R,Pm,H)
    =G= MREQ(R,Pm,H) / SCALE ;

  ASSETMAX(A,Tm)$ASUP(Tm,A) ..
    SUM( (R,H)$COM(A,H), X(A,R,Tm,H)$RCAP(A,R,Tm)
                          / RCAP(A,R,Tm) ) * SCALE
    =L= ASUP(Tm,A) ;
```

```
 THRUPUT(I,Tm)$(PCAP(I,Tm) LT +INF ) ..
   SUM( R$O(R,I), SUM( (A,H)$COM(A,H), X(A,R,Tm,H)$RCAP(A,R,Tm) ))  +
   SUM( R$D(R,I), SUM( (A,H)$COM(A,H), X(A,R,Tm,H)$RCAP(A,R,Tm) ))
   =L=  PCAP(I,Tm) / SCALE ;


*------------------
*  MODEL CONTROL
*------------------

MODEL LIFTOPT /ALL/ ;
OPTION LIMROW=0, LIMCOL=0, SOLPRINT=OFF, ITERLIM=5000;

PARAMETER XA(R,Pm,H,A)     display cargo assigned by aircraft mission
          YA(R,Pm,Po,H)    display late of early movement requirements
          NF(R,Pm,H)       display infeasible movement requirements
          PLANES(R,Pm,A)   display aircraft assignment by mission ;


OPTION XA: 2: 3: 1;
OPTION YA: 2: 3: 1;
OPTION NF: 2: 2: 1;
OPTION PLANES: 2: 2: 1;


*------------------
*  ITWSA CONTROL
*------------------

PARAMETER  ITWSA  determines whether the Independent Time Window  ;
*                 Separation Algorithm is ON or OFF
*                 ( 1=ON; 0=OFF )

          ITWSA = 1;


SET MREQTEST(PM)  terminate zero-MREQ iterations
    BLANKEND(PM)  terminate last zero-MREQ solve
    SOLVTEST(PM)  terminate solve loop when end test satisfied
    PALL(PM)      all Tm if early and late deliveries allowed /T1*T12/;


SCALARS
    ITER      iteration counter
    INFEAS    determines infeasibilities for the current iteration
    CHECK     test for nonzero movement requirement iteration
    TOTLCOST  total cost of all feasible iterations  / 0 /
    SIGN      backward or forward iteration solve discipline
    TEND      ending iteration count number ;

    SIGN = -1$(MAXLTE EQ 0) + 1$(MAXERL EQ 0)
              - 1$( MAXLTE EQ 0 AND MAXERL EQ 0 );
    SIGN =  SIGN$ITWSA ;
    ITER =  CARD(Pm)$(MAXLTE EQ 0) + 1$(MAXERL EQ 0)
              - 1$( MAXLTE EQ 0 AND MAXERL EQ 0 );
    ITER =  ITER$ITWSA ;
```

```
    TEND =  0$(MAXLTE EQ 0) + (CARD(Pm)+1)$(MAXERL EQ 0)
             - (CARD(Pm)+1)$(MAXLTE EQ 0 AND MAXERL EQ 0);
    Tm(Pm)$(SIGN EQ -1)      =  YES $(ORD(Pm) EQ CARD(Pm));
    Tm(Pm)$(SIGN EQ  0)      =  PALL(Pm);
    Tm(Pm)$(SIGN EQ  1)      =  YES $(ORD(Pm) EQ 1);
    TITER(Pm)$(SIGN EQ -1)   =  YES $(ORD(Pm) EQ CARD(Pm));
    TITER(Pm)$(SIGN EQ  1)   =  YES $(ORD(Pm) EQ 1);


*-------------------
* ITWSA SOLVER LOOP
*-------------------

 SOLVTEST(Pm) = YES;
 LOOP(PP$SOLVTEST(PP),

     MREQTEST(PM) = YES$(CARD(TM) LE 1) ;
     BLANKEND(TITER) = YES ;
     LOOP(PL$MREQTEST(PL),
         CHECK     =  1$(SUM((R,Tm,H), MREQ(R,Tm,H))) ;
         ITER      =  ITER + SIGN$(CHECK NE 1) ;
         Tm(Pm)    =  YES$(ORD(Pm) EQ ITER ) ;
         MREQTEST(PM)$(CHECK EQ 1
                       OR  ITER EQ TEND) = NO ) ;

     BLANKEND(TITER)$(SIGN*(ITER - TEND) GE 0) = NO ;
     LOOP(TITER$BLANKEND(TITER),
        SOLVE LIFTOPT USING LP MINIMIZING COST ) ;

     MREQTEST(PM) = YES;
     ITER        =  ITER + SIGN ;
     INFEAS      =  SUM((R,To,H), N.L(R,To,H)) ;
     TOTLCOST    =  TOTLCOST + COST.L$(TOLER GT INFEAS
                                      OR  ITER EQ TEND) ;
     TITER(Pm)   =  YES$(ORD(Pm) EQ ITER ) ;
     Tm(Pm)      =  Tm(Pm)$(INFEAS GT TOLER) + TITER(Pm) ;
     SOLVTEST(PM)$(SIGN*(ITER - TEND) GE 0) = NO  ) ;


    XA(R,Pm,H,A)        =  X.L(A,R,Pm,H)  * SCALE ;
    YA(R,Pm,Po,H)       =  Y.L(R,Pm,Po,H) * SCALE ;
    NF(R,Pm,H)          =  N.L(R,Pm,H) * SCALE ;

    PLANES(R,Pm,A) $ RCAP(A,R,Pm)  =
       SUM( H$COM(A,H), XA(R,Pm,H,A) ) / RCAP(A,R,Pm) ;


    DISPLAY XA, YA, NF, PLANES, TOTLCOST;
```

APPENDIX C. GAMS PROGRAM FOR THE INTEGER ROUNDING MODEL

```
$ONTEXT

        INTEGER AIRCRAFT ROUNDING MODEL  (88.06.01)


        MICHAEL C. PUNTENNEY    -    MODIFIED (89.2.18)

        ACKNOWLEDGEMENT:  THIS MODEL IS BASED ON THE ORIGINAL AIR
                          LIFT ALLOCATION PROBLEM DESCRIBED IN
                          THE TRANSCOM LIFT OPTIMIZER VERSION
                          (87.12.23) BY RICHARD E. ROSENTHAL.

$OFFTEXT

$OFFUPPER  OFFSYMXREF  OFFSYMLIST


*------------------
*  ROUNDING MODULE
*------------------

 PARAMETER  FRAC(A,R,Pm)     fractional PLANE values
            PLAN(R,Pm,A)     rounded plane values for deployment plan ;

    FRAC(A,R,Pm) = PLANES(R,Pm,A) - TRUNC(PLANES(R,Pm,A)) ;


 VARIABLES
    TOTAL          total of fractional values rounded up
    Z(A,R,Pm)      rounded fractional values of PLANES ;

    Z.UP(A,R,Pm) = 1;
    Z.LO(A,R,Pm) = 0;
    Z.FX(A,R,Pm)$(PLANES(R,Pm,A) EQ 0) = 0 ;


 EQUATIONS
    OBJF               objective function
    ALLOCATE(Pm,A)     allocate fractional values observing supplies ;


 OBJF  ..      TOTAL =E= SUM((A,R,Pm), FRAC(A,R,Pm) * Z(A,R,Pm)
                                $(FRAC(A,R,Pm) NE 0)) ;

 ALLOCATE(Pm,A)$ASUP(Pm,A)  ..  SUM(R, Z(A,R,Pm)) =L=
             ASUP(Pm,A) - SUM(R, PLANES(R,Pm,A) - FRAC(A,R,Pm)) ;
```

```
MODEL  ROUNDER /OBJF, ALLOCATE/ ;
OPTION LIMROW=0, LIMCOL=0, SOLPRINT=OFF;

SOLVE ROUNDER USING LP MAXIMIZING TOTAL;


   PLAN(R,Pm,A) = TRUNC(PLANES(R,Pm,A)) + Z.L(A,R,Pm) ;


OPTION PLAN:1:2:1 ;
DISPLAY PLAN ;
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LIST OF REFERENCES

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